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ABSTRACT

This paper focuses on high school preparation for calculus and discusses trends involving student performance, curricula, teachers, and tests. Implications for the transition to calculus in college and the mathematics major are also discussed. (KHR)

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# Trends in High School Preparation for Calculus and Their Implications for the Transition to College

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I discuss here trends involving student performance, curricula, teachers, and tests.

## Trend data on student performance

The most recent data we have on high school student achievement is from the National Assessment of Educational Progress (NAEP). Two performance trends are currently being maintained. A *long-term trend* dates from 1973, and since 1978 the test has used exactly the same items. On this assessment, students are not allowed to use calculators, and the items tested tend to be basic skills considered important in 1973. A *short-term trend* is part of the regular National Assessment administered to 4th graders, 8th graders, and 12th graders in 1990, 1992, 1996, and 2000 (see Table 1). The short-term trend was begun to determine possible effects of the NCTM *Curriculum and Evaluation Standards* (NCTM 1989). Calculators (4-function at grade 4, scientific at grades 8 and 12) are allowed on some sections of the test and are distributed to students who take the test. On both trends, performance of 9-year-olds has increased significantly, by perhaps a grade level since they began. Performance of 13-year-olds also has increased significantly, by perhaps a half grade on the basic skills and by a grade level on the new trend. In slight contrast, performance of 17-year-olds has been steady on basic skills and was about a half grade higher in 2000 than in 1990. These data can be interpreted as showing that, on the average, the basic arithmetic skills of students have not decreased in the past quarter century and students are improving their performance on standards-like questions in the curriculum.

Table 1. National Average Mathematics Scale Scores on the National Assessment of Educational Progress (NAEP), 1990-2000.

| Grade | 1990 | 1992 | 1996 | 2000 |
|---|---|---|---|---|
| 12 | *294 | 299 | *304 | 301 |
| 8 | *263 | *268 | *272 | *275 |
| 4 | *213 | *220 | *224 | *228 |

Source: National Assessment of Educational Progress, Mathematics Report Card 2000, p. 24.

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Why would the greater increases at lower grades on the NAEP not be reflected at higher grades four years later? It could be that the test, being a test of rather basic skills, is not sensitive to the mathematics students encounter at the higher grades. Results from the SATs and ACTs suggest that this reason has some merit, because performance of seniors on both these tests has improved through the 1990s. Mean scores of high school seniors on the mathematics portion of the basic SAT test increased from 501 in 1990 to 516 for last year's senior class (World Almanac 2003, p. 241). Mean scores on the ACT test, taken by over one million seniors in 2000, increased from 19.9 to 20.8 from 1990 to 1998 but have stagnated since 1998 and were 20.6 in 2002 (World Almanac 2003, p. 240). These improvements in mean scores on the ACT and SAT can be considered as an underestimate of an actual increase in mathematics performance by a population comparable with that of 1990 because the percent of the age cohort taking the ACT and SAT has increased a little bit since that time and is now the highest of all time both for the population as a whole [46%] and for the percentage of minority students [35%].

Table 2. Mean mathematics scores of college-bound seniors (SAT) and college-bound students (ACT)

| School year ending | ACT | SAT |
|---|---|---|
| 1975 | | *498 |
| 1980 | | *492 |
| 1985 | | *500 |
| 1990 | *19.9 | 501 |
| 1991 | 20.0 | 500 |
| 1992 | 20.0 | 501 |
| 1993 | 20.1 | 503 |
| 1994 | 20.2 | 504 |
| 1995 | 20.2 | 506 |
| 1996 | 20.2 | 508 |
| 1997 | 20.6 | 511 |
| 1998 | 20.8 | 512 |
| 1999 | 20.7 | 511 |
| 2000 | 20.7 | 514 |
| 2001 | 20.7 | 514 |
| 2002 | 20.6 | 516 |

*ACT tests were reformulated in 1990 and it is not possible to compare with earlier years; SAT tests were "recentered" in 1996 and scores before 1990 are extrapolated on the recentered scale.

Another reason that greater increases at lower grades on the NAEP are not reflected at higher grades four years later might be that middle schools are not taking advantage of the increased knowledge of younger students, and high schools are not taking advantage of the increased knowledge of their entering students. This is certainly possible given the larger numbers of mathematics teachers who are not adequately prepared to teach their subjects. (show transparency)

We should not be surprised that in the year 2000, the most recent year for which we have data, only 26% of high school mathematics teachers in a randomly-selected national sample felt well-qualified to teach statistics and only 41% felt well-qualified to teach probability (see Table 3). Even one course in statistics is still not required in many mathematics departments for a bachelor's degree in mathematics. However, it has to give us pause when only 70% of high school mathematics teachers feel very well qualified to teach geometry and spatial sense, and only 61% feel very well qualified to teach functions and pre-calculus concepts. Algebra is the only subject that over 90% of teachers feel very well qualified to teach, but how can you teach that well if you do not also know functions well?

Table 3. High School Mathematics Teachers' Perceptions of Their Qualifications to Teach Each of a Number of Mathematics Subjects

| Subject | Not Well | Adequately | Very Well |
|---|---|---|---|
| Algebra | 0 | 5 | 94 |
| Geometry & Spatial Sense | 4 | 26 | 70 |
| Functions & Precalculus | 6 | 33 | 61 |
| Probability | 10 | 49 | 42 |
| Statistics | 23 | 51 | 26 |
| Calculus | 39 | 37 | 25 |
| Topics from Discrete Math | 44 | 40 | 16 |

Source: Dawayne Whittington, Status of High School Mathematics Teaching, from the 2000 National Survey of Science and Mathematics Education. Chapel Hill, NC: Horizon Research, Inc. (December 2002). www.horizon-research.com

**Variability within the United States**

The variability between states and other large jurisdictions is quite a bit larger than the increases even within a decade on the NAEP. Using NAEP estimates for comparing their scaled score, the typical 8th grader in North Dakota is approximately 3 years ahead of the typical 8th grader in Mississippi, and the typical 8th grader in Washington DC seems to be at about the same level in mathematics as the typical 4th grader in the rest of the country. A different sort of analysis has been done matching jurisdictions to scores on the Third

International Mathematics and Science Study. Top-performing collections of school districts in the U.S. have scored as well as the highest-performing countries in the world; lower-performing states have scored as low as third-world countries.

These differences have major implications for policy decisions. A "one size fits all" policy decision for the nation is easy to defend by a theoretical argument appealing to democracy and equal opportunity for all, but the actual differences between schools make a single policy fundamentally unwise.

Differences in performance through the U.S. are mirrored in startling differences of graduation rates. In some public schools, virtually 100% of students expect to go to college, while in others fewer than half the students graduate.

This in itself has major implications for the preparation of students for college-level mathematics. Where fewer students graduate high school, fewer go on to college. And where fewer go on to college, it is more difficult to teach a college-preparatory curriculum. Consequently, differences in student graduation rates have major implications for the high school mathematics teacher and the high school mathematics curriculum.

Schools in which almost all students go to college are likely to have advanced placement calculus courses and high percents of students taking precalculus or calculus courses by the time they graduate. In these schools, calculus is perceived by both students and teachers as a first-year college subject that is important for a diverse collection of fields of study and that might be appropriate for any well-prepared student. But in high schools where a small percentage of students graduate, calculus is rarely taught and enrollments in precalculus courses are low. Calculus is viewed as a course well beyond high school, seldom taken even by college freshmen, a mysterious course that is hard and taken only by those particularly interested in mathematics or science.

Another circumstance that affects student views towards calculus is that 37% of the 1.3 million seniors who took the SAT last year are first-generation college students (College Board 2002). This is a substantially higher percentage than even a year ago (28%). These students are likely to view calculus as more arcane than other students whose parents went to college.

**High school mathematics curricula**

Sometimes we need to be reminded that mathematics is not the only subject taken by high school students, and academic subjects are not the only things vying for a student's homework time. English, social studies, science, and foreign language departments all want students to take 3-4 years of their subjects. Consequently, a student desirous of getting into a selective school either takes 5 major subjects most years or goes to summer school. Students are also encouraged to participate in sports, music, or any of a variety of school clubs or other activities. Furthermore, more than two-thirds of 12th-grade students taking the NAEP test in 2000 reported having a part-time job, with 56% reporting working more than 10 hours

a week, and 27% reporting working more than 20 hours a week. Even ignoring family, a social life, and the world in general, many things vie for the time and attention of a typical high school student.

The percents of students taking the various college preparatory mathematics courses have been rising steadily for 20 years (see Dossey and Usiskin 2000, p. 15 and Table 4).

Table 4. Highest mathematics course taken by high school graduates in the United States, from transcript studies in selected years

| Course‡ | 1982 | 1987 | 1990 | 1994 | 1998 |
|---|---|---|---|---|---|
| Pre-algebra or General Math.* | 25.0 | 20.6 | 17.9 | 13.1 | 10.1 |
| Alg 1 or Geom or Unified 1,2 | 30.6 | 26.8 | 25.4 | 22.4 | 20.8 |
| Algebra 2 or Unified 3 | 18.2 | 23.1 | 26.2 | 26.4 | 27.7 |
| Algebra/trig | 15.5 | 12.9 | 12.9 | 16.3 | 14.4 |
| Precalculus | 4.8 | 9.0 | 10.4 | 11.6 | 15.2 |
| Calculus | 5.9 | 7.6 | 7.2 | 10.2 | 11.8 |

Source: National Center for Education Statistics. *The Condition of Education 2002*. Washington, DC: U.S. Government Printing Office, 2002, pp. 157, 215-6.

‡ These studies considered more descriptors of the courses than the most commonly used descriptors named here.

* Entries in this row were found by subtracting the sum of other entries in each column from 100.

Similar percentages gathered from reports of students taking the SAT are even higher. Last year, 25% of seniors taking the SAT reported taking calculus before graduation, and 45% reported taking precalculus, up from 20% for calculus and 33% for precalculus ten years ago. The cluster of data on enrollments may be rife with overestimates but yields a robust conclusion: students are taking more mathematics now than they did ten years ago.

What content is covered in these courses? I distinguish the traditional high school mathematics curriculum from others. The traditional high school mathematics curriculum emphasizes algebra and functions, de-emphasizes Euclidean geometry except as an important vehicle for learning about proof, and ignores statistics almost completely. From pre-algebra in the middle school through second-year algebra, much time is spent having students do algebraic manipulations of the type that are used in calculus, such as solving linear and quadratic equations and systems, and factoring and performing operations on polynomials. Functions are introduced starting in the second year of algebra and are a main focus of study through the remainder of the precalculus experience. Almost all the content of the traditional high school mathematics curriculum is designed to prepare students for calculus.

The percents of students taking geometry and second-year algebra are now greater than the percent that will go on to college. This has an obvious effect on what teachers feel their students can successfully learn. Teachers can adjust either by easing the course somewhat or by broadening its content to be of interest, or they can keep the course the same and fail a significant number of students. Most teachers adjust the course, even if only slightly. So I believe less time is being spent on complicated algebraic manipulations involving rational expressions or radicals than used to be the case, and less on proof in geometry, even though this material is in the texts from which they teach. This also may be due to the influence of the NCTM *Curriculum and Evaluation Standards* (NCTM 1989).

The newer curricula, those that follow the guidelines of the NCTM Standards (NCTM 1989, 2000), have clearly been influenced by these increased enrollments. They are designed not only to prepare students for *any* of the first mathematics courses students might take in college – calculus, statistics, computer science, or finite mathematics – but also to appeal to students who may not continue their education. They give significant attention to mathematical modeling and statistics, de-emphasize the proof aspect of Euclidean geometry while concentrating on properties of measurement, and they discuss algorithms and discrete functions. They tend to downplay abstract work with polynomials, rational expressions, and radicals (e.g., see Core Plus 1997 and IMP 1998).

A common rhetoric is found in discussions of pre-calculus and calculus reform at the college level and discussions of reform of the high school curriculum. Broadening the mathematics experience by using real-world data and modeling, utilizing the latest technology, involving students more in their own learning, and downplaying manipulative aspects and proof are some of the commonalities. Accordingly, the same tension that has existed at the college level between traditional and reform calculus exists at the high school level between traditional and newer curricula.

Calculators are used in all curricula. On the 2000 NAEP, 69% of 12th graders reported using a calculator every day, while only 10% reported never or hardly ever using a calculator (Braswell et al., p. 159). Mean scores of students increased significantly with frequency of calculator use. On the teacher study that I cited earlier, 79% of teachers reported students using calculators in their most recent lesson, and graphing calculators were used in 94% of classes beyond geometry (Whittington, pp. 19, 23).

**Calculus at the high school level**

In 2001, 5% of the graduating seniors in the country had taken the AB calculus exam and 1.3% had taken the BC calculus exam. These are all-time records. Since 18% of all 12th graders on the 2000 NAEP reported being enrolled in a calculus course, less than 1 in 3 students enrolled in calculus in high school took the AP exam. The majority seem to be taking the course in high school in order to show colleges by their transcripts that they are taking difficult courses and to increase their chances of getting a high grade in college

calculus. That is, if they ever get to calculus. The data collected 15 years ago by Waits and Demana (1988) may still apply today. They found that only about 28% of the [1721] freshmen who entered Ohio State University in 1986 with *five* or more years of college-preparatory mathematics were ready for calculus. For Ohio State, readiness for calculus was determined then by a placement test that had "remained essentially unchanged for the past twenty-five years" (Waits and Demana 1988, p. 11).

Data such as that collected by Waits and Demana are often used by college mathematics departments to discourage students from studying calculus before college. The argument is that the teaching of calculus in high schools is poor, often done by individuals unqualified to teach the subject, and results in students learning concepts in wrong ways. I disagree for two reasons. First, the environment in which calculus is taught is far better in high schools than in colleges. The teachers know their students personally, having seen many of them in their courses in prior years, and they care about them. The teachers are typically the best-qualified mathematically and among the most experienced and able teachers in the school. Also, the students know and help each other in a familiar and comfortable setting. Second, when the same argument is (in my opinion, wrongly) offered to discourage the teaching of algebra before high school it is found to be riddled with holes. Students need to have studied some algebra before high school in order to be successful in the typical high school algebra course. There is simply too much material to digest to expect a student to progress from never having worked with variables to the study of linear systems, quadratic equations, radicals, and rational expressions in a single year. Perhaps the most significant reason that calculus is so difficult for many students is that we try to teach it from scratch in a single year.

This suggests that, before their calculus course, students should encounter *multiple times* the topics of inequality, distance and areas on the coordinate plane, area, rate and rate of change, infinity, sequence, function, limit, max-min problems, and summation as part of their experiences with algebra, geometry, functions, statistics, and discrete mathematics. It also suggests that students have at least one introduction to derivatives and integrals. However, because calculus is not the only area of college-level mathematics for which students need some background, I believe that K-8 curricula should be designed so that algebra is taken by most students in 8th grade not because it makes it possible for them to take a calculus course in high school, but because it provides an extra year to prepare them for calculus and the other mathematics they are likely to encounter in college. Likewise, in an optimal calculus experience students should revisit some algebra, geometry, probability, and statistics from earlier years, and also be introduced to concepts of differential equations, complex variables, and algebraic structures that they might encounter in later years. College students' lack of exposure to the latter topics in early undergraduate mathematics courses is surely one of the reasons they have difficulty with them in later undergraduate mathematics.

**Teachers and Tests**

If average performance is going up, and if students are taking more courses, why do students throughout the nation perform so poorly on college placement tests? Factors other than lack of competence contribute. As had to be the case with Ohio State's 25-year-old test in 1986, placement tests often ignore much of the newer content and many of the newer ways of teaching, such as: working with a variety of representations of functions, including graphical and tabular approaches; modeling and otherwise dealing with data; and using the technology of calculators and computers as a helpful tool. There exist no review classes for most placement tests as there do for the SAT and ACT. Often there are no sample exams nor a detailed syllabus given to students from which to study. High school teachers cannot prepare their students for a particular placement exam because their students go to a variety of institutions. And the placement tests are often taken under conditions that are far from optimal. They are given to students who may be away from home for the first time, who may have just had a medical exam or come from long waits for ID cards, and who may have stayed up late the night before talking to other freshmen in their dorms. And large numbers of students are taking these exams who, in previous decades, would not have had to take them or would not have attended college.

Most high school mathematics teachers realize that many of their students will take some statistics and computer science in college, and that many will enter the health professions (currently the most popular major reported by students taking the SAT) or become business majors – all areas that require calculus, but not necessarily a mathematician's calculus. Students are well aware that computers and calculators are universally available outside the classroom but that tests may not allow them to use this technology. Faced with a diverse group of students coming in who will be going out to a diverse set of institutions or to various workplace settings, high school mathematics departments and their teachers are faced with difficult decisions regarding what curricula and technology to use.

These decisions are made more difficult because the various high-stakes external tests have quite different goals. The SATs and ACTs allow graphing calculators. Many college placement tests disallow all calculators. Some college placement tests emphasize symbolic manipulation; others don't. Few touch upon statistics or geometry. Parts or all of these tests may conflict with the teacher's or the school mathematics department's own tests to determine student grades. Yet the teacher is under pressure to ensure that students perform well on all of these tests. Thus the high school teacher is beleaguered, faced with pressures from inside and outside the education profession to get students to perform at high levels with and without calculators, on traditional content and newer content. They are expected to do this with a large population of students whose parents never studied the courses and thus have no idea why this mathematics is needed.

For many students, the college mathematics placement test carries with it extraordinarily high stakes. The test can keep a person from calculus for one or two years, thereby greatly influencing the majors possible for that person. Yet these tests receive little publicity and are accountable to no one. A national effort involving all the mathematical sciences and other disciplines we serve, and high school and college mathematics educators is needed to bring these tests in line so that we can reward rather than penalize today's students for the wider range of knowledge and techniques which they bring to their overall study of mathematics.

**Calculus in college and the mathematics major**

That we have a session devoted to preparation for calculus is a sign of the importance of calculus itself. High school and college mathematics faculties both tend to view calculus as the first college course in mathematics. But, for students, calculus is very much the culmination of five or more years of study. Much of what they have studied in mathematics has been justified by teachers by "You will need this for calculus." Many students believe that when they study calculus, they will finally learn what mathematics is all about, how mathematics explains the universe, and how to solve all sorts of problems.

This is too high a burden to place on any single course. Traditional calculus may have mathematical sanctity, but it rarely yields the epiphany students hope for. I recognize that the reform calculus movement has been motivated in part by a desire to attack this problem, but I wish to take the argument beyond calculus. This may be pushing the envelope in a session devoted to preparation for calculus, but one cannot examine preparation for calculus without examining all of the roles of calculus itself.

Why is the number of mathematics majors in the country declining even though high schools are turning out record numbers of students who have been successful in calculus? One possible reason is that calculus has lost its role as the first course for mathematics majors and cannot have that role for students who take it in high school. Consequently, college mathematics departments need a broad-based post-calculus course for committed and potential mathematics majors to induce them into majoring in the mathematical sciences. Such a course should not assume that all smart people believe mathematics is beautiful, important, and useful, nor that all mathematics majors become either teachers or research mathematicians. Discussions *about* mathematics should be part of the course's agenda.

**References**


Braswell, James S., Anthony D. Lutkus, Wendy S. Grigg, Shari L. Santapau, Brenda Tay-Lim, and Matthew Johnson. *The Nation's Report Card: Mathematics 2000.* Washington, DC: National Center for Education Statistics, Office of Educational Research and Improvement, U.S. Department of Education, 2001.

Campbell, Jay R., Kristin Voelkl, and Patricia Donahue. *NAEP 1996 Trends in Academic Progress.* Washington, D.C.: National Center for Education Statistics,1997.

College Board. "10-year Trend in SAT® Scores Indicates Increased Emphasis on Math is Yielding Results; Reading and Writing are Causes for Concern", press release, August 27, 2002 http://www.collegeboard.com

Core-Plus Mathematics Project. *Contemporary Mathematics in Context: A Unified Approach.* Courses 1-3. Chicago, Ill.: Everyday Learning, 1997.

Dossey, John, and Zalman Usiskin. *Mathematics Education in the United States 2000.* Reston, VA: National Council of Teachers of Mathematics, 2000.

Henry, Tamara. "Advanced Placement faces crossroads." *USA Today*, 26 Sept 2001, p. 6D.

Interactive Mathematics Program (IMP). *Introduction and Implementation Strategies for the Interactive Mathematics Program.* Berkeley, Calif.: Key Curriculum Press, 1998.

Mullis, Ina V. S., Michael O. Martin, Albert E. Beaton, Eugenio J. Gonzalez, Dana L. Kelly, Teresa A. Smith. *Mathematics Achievement in the Primary School Years.* Boston, Mass.: TIMSS International Study Center, 1997.

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM 1989.

_____. *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

University of Chicago School Mathematics Project (UCSMP). *Transition Mathematics* (2002). *Algebra* (2002). *Geometry* (2002). *Advanced Algebra* (2002). *Functions, Statistics, and Trigonometry* (1997). *Precalculus and Discrete Mathematics* (1998).

U.S. Census Bureau. http://www.census.gov/population (September 2001).

Usiskin, Zalman. Recent Developments in Secondary School Mathematics, and Their Implications. In Anita Solow (Ed.), *Preparing for a New Calculus.* MAA Notes No. 36. Washington, DC: The Mathematical Association of America, 1994.

_____. Algebra and Calculus for All? UCSMP Newsletter No. 18 (Winter 1996); also in the *Illinois Mathematics Teacher* (September 1996) and the *Journal of Mathematics and Science: Collaborative Explorations* (Spring 1999).

_____. High School Overview and the Transition to College. In *Proceedings of a Conference on the Preparation for Calculus* (tentative title). Washington, DC: Mathematical Association of America (to appear).

Waits, Bert K., and Franklin Demana. Is Three Years Enough? Mathematics Teacher 81:1 (January 1988), 11-15.

Wirtz, Willard, et al. *On Further Examination. Report of the Advisory Panel on the Scholastic Aptitude Test Score Decline*. New York: College Entrance Examination Board, 1977.

*The World Almanac and Book of Facts 2003*. New York: World Almanac Books, 2002.

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